- Shaking Force – The sum of all forces acting on the ground plane

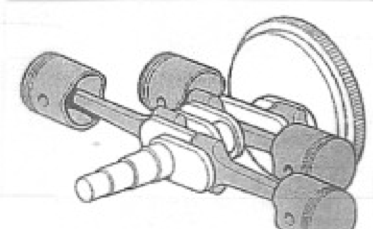
$$F_s = F_{21} + F_{41}$$

- Shaking Torque – The reaction torque felt by the ground plane

$$T_s = T_{21}$$ where T_{12} = source torque delivered to the driving link from the ground

Balancing Linkages

Complete balance of any mechanism can be obtained by creating a second "mirror image" mechanism connected to it so as to cancel all dynamic forces and moments. This approach is expensive and only justified if the added mechanism serves some purpose (increasing power).



(a) Opposed four

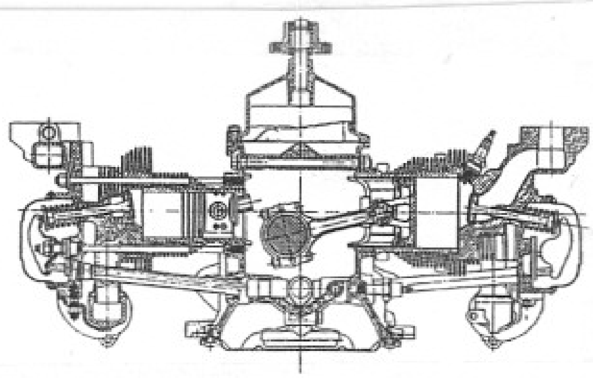


FIGURE 14-6

Chevrolet Corvair horizontally opposed six-cylinder engine
Courtesy of Chevrolet Division, General Motors Corp.

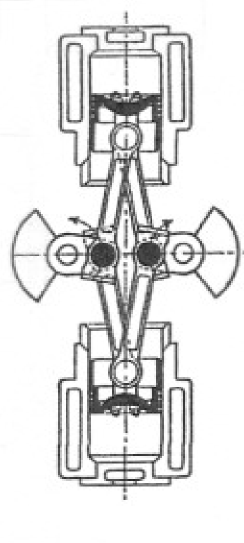
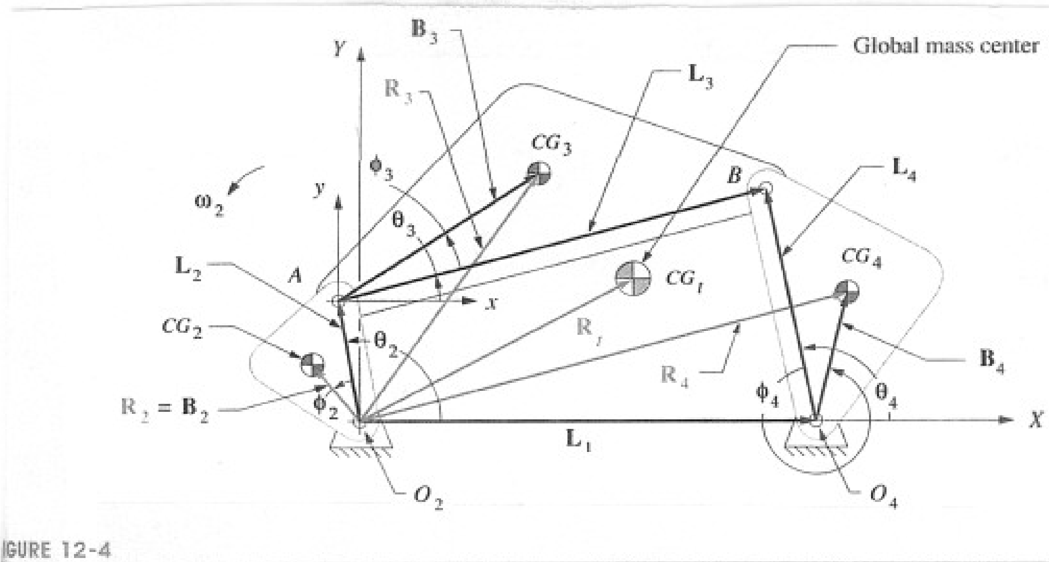


FIGURE 14-32

Perfectly balanced 1897 Lanchester two-cylinder engine

- Several methods have been created to balance linkages. It is difficult to completely balance one dynamic factor (shaking force) while eliminating the others (shaking moment and driving torque).

- For a four bar linkage, the rotating links (crank and rocker) can be balanced using the balancing methods we described in lectures 18 and 19. But the coupler has no fixed pivot and thus its mass center is, in general, always in motion.



IGURE 12-4
atic (force) balancing a fourbar linkage

- Any mechanism will have a single global mass center located at some particular point. If we can force the global mass center to remain stationary we will have a state of static balance for the overall linkage.
- The Berkof - Lowen method provides a way to calculate the magnitude and location of counterweights to be placed on the rotating links, which will make the global mass center stationary for all positions of the linkage. →

→ Placement of the proper balance masses on the links will cause the dynamic forces on fixed pivots to always be equal and opposite. The linkage will be statically forced balanced ($\Sigma F = 0$) but the skaking torque will not be balanced ($\Sigma M \neq 0$).

- The position vectors of the CGs are $\vec{R}_2$, $\vec{R}_3$, $\vec{R}_4$ and $\vec{R}_t$

The product of the CG radius and the mass is

$$m_t \vec{R}_t = m_2 \vec{R}_2 + m_3 \vec{R}_3 + m_4 \vec{R}_4 \qquad (1)$$

rearranging → $$\vec{R}_t = \frac{m_2 \vec{R}_2 + m_3 \vec{R}_3 + m_4 \vec{R}_4}{m_t}$$

We would like to force $\vec{R}_t$ to be constant

From the linkage geometry

$$\vec{R}_2 = b_2 e^{j(\theta_2 + \phi_2)} = b_2 e^{j\theta_2} e^{j\phi_2}$$

$$\vec{R}_3 = l_2 e^{j\theta_2} + b_3 e^{j(\theta_3 + \phi_3)} = l_2 e^{j\theta_2} + b_3 e^{j\theta_3} e^{j\phi_3}$$

$$\vec{R}_4 = l_1 e^{j\theta_1} + b_4 e^{j(\theta_4 + \phi_4)} = l_1 e^{j\theta_1} + b_4 e^{j\theta_4} e^{j\phi_4}$$

Now substitute $\vec{R}_2$, $\vec{R}_3$ and $\vec{R}_4$ into Eq. (1)

$$m_t \vec{R}_t = \left(m_4 l_1 e^{j\theta_1}\right) + \left(m_2 b_2 e^{j\phi_2} + m_3 l_2\right) e^{j\theta_2} + \left(m_3 b_3 e^{j\phi_3}\right) e^{j\theta_3} + \left(m_4 b_4 e^{j\phi_4}\right) e^{j\theta_4} \qquad (2)$$

The terms in parentheses are all constant with time.

We can also write the loop closure equation

$$l_2 e^{j\theta_2} + l_3 e^{j\theta_3} - l_4 e^{j\theta_4} - l_1 e^{j\theta_1} = 0$$

solving for the link 3 unit vector

$$e^{j\theta_3} = \frac{l_1 e^{j\theta_1} - l_2 e^{j\theta_2} + l_4 e^{j\theta_4}}{l_3}$$

Now substitute back into Equation (2) and collect terms

$$m_t \vec{R}_t = \left(m_2 b_2 e^{j\phi_2} + m_3 l_2 - m_3 b_3 \frac{l_2}{l_3} e^{j\phi_3}\right) e^{j\theta_2} +$$

$$\left(m_4 b_4 e^{j\phi_4} + m_3 b_3 \frac{l_4}{l_3} e^{j\phi_3}\right) e^{j\theta_4} +$$

$$+ m_4 l_1 e^{j\theta_1} + m_3 b_3 \frac{l_1}{l_3} e^{j\phi_3} e^{j\phi_1}$$

The only terms that are time varying are $e^{j\theta_2}$ and $e^{j\theta_4}$ the rest are constant. Therefore if we want $m_t \vec{R}_t$ to be a constant the terms in parentheses must be made zero

$$\therefore \quad m_2 b_2 e^{j\phi_2} + m_3 l_2 - m_3 b_3 \frac{l_2}{l_3} e^{j\phi_3} = 0$$

$$m_4 b_4 e^{j\phi_4} + m_3 b_3 \frac{l_4}{l_3} e^{j\phi_3} = 0$$

Rearranging terms

$$m_2 b_2 e^{j\phi_2} = m_3 \left(b_3 \frac{l_2}{l_3} e^{j\phi_3} - l_2\right)$$

$$m_4 b_4 e^{j\phi_4} = -m_3 b_3 \frac{l_4}{l_3} e^{j\phi_3}$$

Now let's break it up into x and y components →

$$(m_2 b_2)_x = m_3 \left(b_3 \frac{l_2}{l_3} \cos\phi_3 - l_2 \right)$$

$$(m_2 b_2)_y = m_3 \left(b_3 \frac{l_2}{l_3} \sin\phi_3 \right)$$

$$(m_4 b_4)_x = - m_3 b_3 \frac{l_4}{l_3} \cos\phi_3$$

$$(m_4 b_4)_y = - m_3 b_3 \frac{l_4}{l_3} \sin\phi_3$$

These four terms represent the components of the mR product needed to force balance the linkage

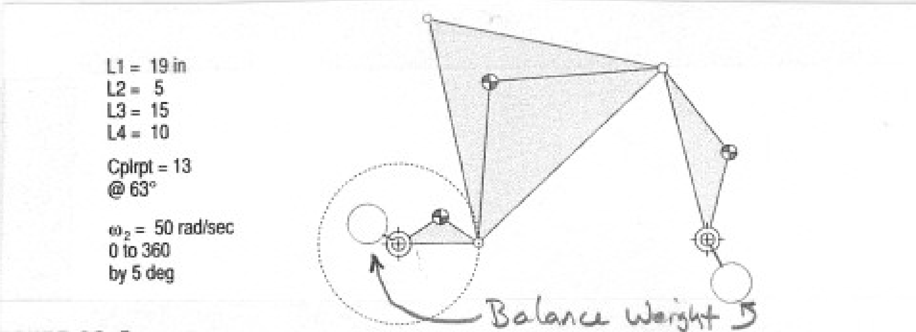
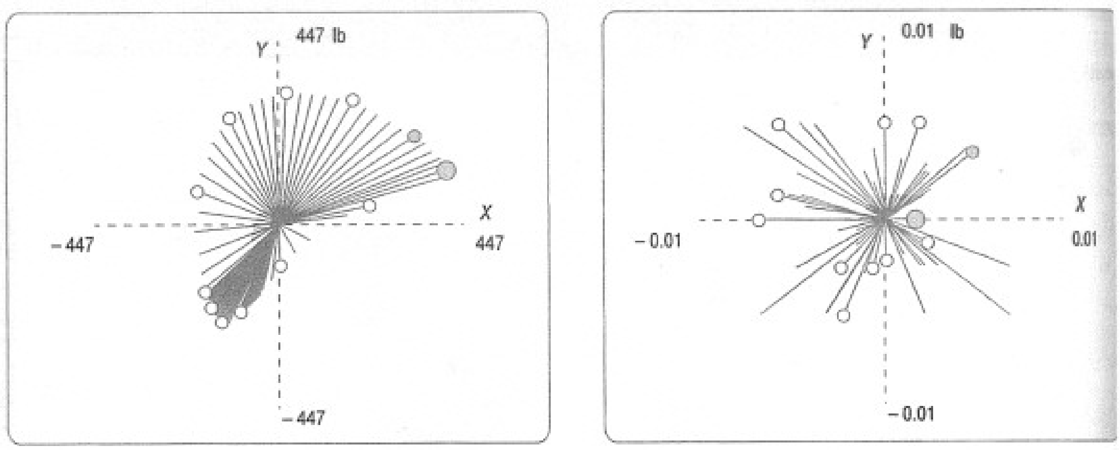


FIGURE 12-5

A balanced fourbar linkage showing balance masses applied to links 2 and 4

* Open the disk file F12-05.4br in program FOURBAR to see more detail on this linkage and its balancing.

There is a considerable reduction in the shaking force after the linkage has been balanced.



(a) Shaking force with linkage unbalanced

(b) Shaking force with linkage balanced

FIGURE 12-6

Polar plot of unbalanced shaking forces on ground plane of the fourbar linkage of Figure 12-5

Note: The forces are balanced but not the shaking torque.